The economics of sharing the revenues from sportscast*

Gustavo Bergantiños[†]

Juan D. Moreno-Ternero[‡]

July 4, 2023

Abstract

Sports are one of the most significant products of the entertainment industry, accounting for a large portion of all television viewing. Consequently, the sale of broadcasting and media rights is the most important source of revenue for professional sports clubs. We survey the economic literature dealing with this issue, with a special emphasis on the crucial problem that arises with the allocation of revenues when they are raised from the collective sale of broadcasting rights.

***JEL numbers**: D63, C71, Z20.*

***Keywords**: sportscast, broadcasting rights, sport leagues, resource allocation, axioms.*

*Financial support from the Ministerio de Ciencia e Innovación through the research projects PID2020-115011GB-I00, and PID2020-113440GB-I00 (funded by MCIN/AEI/ 10.13039/501100011033), Xunta de Galicia through grant ED431B 2022/03 and Junta de Andalucía through grant A-SEJ-14-UGR20 is gratefully acknowledged.

[†]ECOBAS, Universidade de Vigo, ECOSOT, 36310 Vigo, Spain.

[‡]Department of Economics, Universidad Pablo de Olavide, 41013 Seville, Spain

1 Introduction

On December 18, 2022, nearly 1.5 billion viewers watched the final game of the FIFA World Cup Qatar 2022. Ten professional sports leagues worldwide (Ligue 1, Serie A, Bundesliga, UEFA Champions League, La Liga, Indian Premier League, National Basketball Association, Major League Baseball, National Football League and English Premier League) have at least 1 billion viewers. These viewers constitute a source of massive revenues, crucial for the management of sports organizations. This was exemplified in the huge (and somewhat controversial) efforts to resume competitions worldwide in the aftermath of the COVID-19 pandemic's first wave, in order to secure broadcasting contracts (in spite of having empty stadiums).

Although a rising concern for the sports industry is the behavior of young consumers, who have grown up in a digital world with abundant access to free content, broadcasting contracts still offer sizable amounts. For instance, the N.F.L. signed new media rights agreements with CBS, NBC, Fox, ESPN and Amazon collectively worth about $110 billion over 11 years, to take effect in 2023, nearly doubling the value of its previous contracts. Almost simultaneously, the English Premier League confirmed it would extend its television deal, obtaining more than £10 billion for each of the three upcoming seasons.

It is well known that bundling products may increase revenue with respect to selling products independently (e.g., Adams and Yellen, 1976) and real-life instances in which bundling occurs abound (e.g., Bergantiños and Moreno-Ternero, 2015). This rationalizes the sale of broadcasting rights for sports leagues is often carried out through some sort of collective bargaining.[1]

The ensuing sharing process among participating teams is a complex problem and sharing rules vary across the world. For instance, in North America, contracts essentially involve equal sharing, whereas in Europe, performance-based reward schemes are widespread, which is rationalized by the fact that European leagues compete for talent (e.g., Palomino and Szakovics, 2004).[2] But even within European leagues rules vary considerably. For instance, FC Barcelona and Real Madrid CF, the two Spanish giant football teams, used to earn each more than 20% of the revenues generated by the Spanish Football League (e.g., Bergantiños and Moreno-Ternero, 2020a). In England, however, the two teams earning more only made together 13% of the revenues generated by the English Premier League. This might partly explain why in the last 19 editions of the Spanish Football League only twice the champion was neither FC Barcelona nor Real Madrid CF (Atletico de Madrid), whereas the Premier League witnessed 4 different champions (Manchester United, Manchester City, Leicester and Chelsea) in the 4 editions from 2013 to 2016 (and Liverpool won in 2020).

The rest of the survey is organized as follows. In Section 2, we review the axiomatic approach to the problem. In Section 3, we review the game-theoretical approach to the problem. In Section 4, we review a statistical estimation approach. In Section 5, we review the decentralized approach via voting. In Section 6, we review extensions of the model via the operational approach. In Section 7, we review the empirical applications. All these sections include open questions for future research. Finally, we conclude in Section 8.

[1] Falconieri et al., (2004) provide a welfare analysis of collective vs. individual sale of TV rights. Peeters (2012) studies how media revenue sharing acts as a coordination device in sports leagues.

[2] North America's one-team-one-vote environment paves the way for equal sharing as the national contract can be approved only if there is a virtual consensus among league teams (e.g., Fort and Quirk, 1995).

2 An axiomatic approach

We consider the benchmark model introduced in Bergantiños and Moreno-Ternero (2020a). Let N be a finite set of teams. Its cardinality is denoted by n. We assume $n \geq 3$. For each pair of teams $i, j \in N$, we denote by a_{ij} the broadcasting audience (number of viewers) for the game played by i and j at i's stadium. We use the notational convention that $a_{ii} = 0$, for each $i \in N$. Let $A \in \mathcal{A}_{n\times n}$ denote the resulting matrix of broadcasting audiences generated in the whole tournament involving the teams within N.[3] As the set N will be fixed throughout our analysis, we shall not explicitly consider it in the description of each problem. Each matrix $A \in \mathcal{A}_{n\times n}$ with zero entries in the diagonal will thus represent a *problem* and we shall refer to the set of problems as $\mathcal{P}$.

Let $\alpha_i(A)$ denote the overall audience achieved by team i, i.e.,

$$\alpha_i(A) = \sum_{j\in N}(a_{ij} + a_{ji}).$$

For ease of exposition, we normalize the revenue generated from each viewer to 1 (to be interpreted as the "pay per view" fee). Thus, we sometimes refer to $\alpha_i(A)$ as the *claim* of team i. When no confusion arises, we write α_i instead of $\alpha_i(A)$. We then denote each team's (overall) home audience by h_i and its (overall) away audience by w_i. Formally, for each $i \in N$,

$$\begin{aligned} h_i &= \sum_{j\in N\setminus\{i\}} a_{ij}, \text{ and} \\ w_i &= \sum_{j\in N\setminus\{i\}} a_{ji}. \end{aligned}$$

Note that $\alpha_i = h_i + w_i$, for each $i \in N$.

We denote by $\overline{\alpha}$ the average audience of all teams. Namely,

$$\overline{\alpha} = \frac{\sum_{i\in N}\alpha_i}{n}.$$

For each $A \in \mathcal{A}_{n\times n}$, let $||A||$ denote the overall audience of the tournament. Namely,

$$||A|| = \sum_{i,j\in N} a_{ij} = \frac{1}{2}\sum_{i\in N}\alpha_i = \frac{n\overline{\alpha}}{2}.$$

A **rule** is a mapping that associates with each problem the list of the amounts teams get from the overall revenue. Formally, $R : \mathcal{P} \to \mathbb{R}^N$ is such that, for each $A \in \mathcal{P}$,

$$\sum_{i\in N} R_i(A) = ||A||.$$

In this section, we review the axiomatic approach to derive specific rules. That is, rather than proposing rules directly, the focus is on formalizing axioms that reflect properties for those rules with normative appeal. Several combinations of some of those axioms will eventually lead towards specific rules.

[3] We are therefore assuming a round-robin tournament in which each team plays in turn against each other team twice: once home, another away. This is the usual format of the main European football leagues. Our model could also be extended to leagues in which some teams play other teams a different number of times and play-offs at the end of the regular season, which is the usual format of North American professional sports. In such a case, a_{ij} is the broadcasting audience in all games played by i and j at i's stadium.

We start presenting two fundamental axioms with a long tradition in axiomatic work. The first one is a structural axiom that says that revenues should be additive on A. Formally,

Additivity: For each pair A and $A' \in \mathcal{P}$,

$$R\left(A+A'\right)=R(A)+R\left(A'\right).$$

Second, an axiom indicating that the name of the agents does not matter. Formally, let σ be a permutation of the set of agents. Thus, $\sigma : N \to N$ such that $\sigma\left(i\right) \neq \sigma\left(j\right)$ when $i \neq j$. Given a permutation σ and $A \in \mathcal{P}$, we define the problem A^{σ} where, for each pair $i, j \in N$, $a_{ij}^{\sigma} = a_{\sigma(i)\sigma(j)}$.

Anonymity: For each $A \in \mathcal{P}$, each permutation σ, and each $i \in N$,

$$R_i(A) = R_{\sigma(i)}(A^{\sigma}).$$

As the next result states the two axioms together characterize a general family of rules structured in the following way. Assume the amount received by each team i has three parts: one depending on its (overall) home audience, another depending on its (overall) away audience, and the third depending on the overall audience in the whole tournament. Formally,

General rules $\{G^{xyz}\}_{x+y+nz=1}$. For each trio $x, y, z \in \mathbb{R}$ with $x+y+nz=1$, each $A \in \mathcal{P}$, and each $i \in N$,

$$G_i^{xyz}(A) = xh_i + yw_i + z\left|\left|A\right|\right| = x\left(h_i - \frac{\bar{\alpha}}{2}\right) + y\left(w_i - \frac{\bar{\alpha}}{2}\right) + \frac{\bar{\alpha}}{2}.$$

Theorem 1 *(Bergantiños and Moreno-Ternero, 2023a) A rule satisfies additivity and anonymity if and only if it is a general rule.*

An interesting sub-family of *general* rules arises from compromising between two focal rules: the so-called *uniform* rule, which splits evenly the overall amount among all participating teams, and *concede-and-divide*, which rewards teams comparing their individual performance with the average performance of the remaining teams in the tournament.[4] Formally,

Uniform, U: for each $A \in \mathcal{P}$, and each $i \in N$,

$$U_i(A) = \frac{||A||}{n} = \frac{\overline{\alpha}}{2}.$$

Concede-and-divide, CD: for each $A \in \mathcal{P}$, and each $i \in N$,

$$CD_i(A) = \alpha_i - \frac{\sum\limits_{j,k \in N\backslash\{i\}} (a_{jk} + a_{kj})}{n-2} = \frac{(n-1)\,\alpha_i - ||A||}{n-2} = \frac{2\,(n-1)\,\alpha_i - n\overline{\alpha}}{2(n-2)}.$$

Compromise rules $\left\{UC^{\lambda}\right\}_{\lambda \in \mathbb{R}}$: for each $\lambda \in \mathbb{R}$ each $A \in \mathcal{P}$, and each $i \in N$,

$$UC_i^{\lambda}(A) = (1-\lambda)U_i(A) + \lambda CD_i(A).$$

Equivalently,

$$UC_i^{\lambda}(A) = (1-\lambda)\frac{||A||}{n} + \lambda\frac{(n-1)\,\alpha_i - ||A||}{n-2} = \frac{\overline{\alpha}}{2} + \lambda\frac{n-1}{n-2}\left(\alpha_i - \overline{\alpha}\right).$$

[4] For ease of exposition, we use a slightly different terminology to that in Bergantiños and Moreno-Ternero (2021, 2022a).

It turns out that if we replace *anonymity* by *equal treatment of equals* (another impartiality axiom stating that if two teams have the same audiences each time they play a third, then they should receive the same amount) in Theorem 1, then the family of compromise rules is characterized.[5] Formally,

Equal treatment of equals: For each $A \in \mathcal{P}$, and each pair $i, j \in N$ such that $a_{ik} = a_{jk}$, and $a_{ki} = a_{kj}$, for each $k \in N \setminus \{i, j\}$, $R_i(A) = R_j(A)$.

Theorem 2 *(Bergantiños and Moreno-Ternero, 2022a) A rule satisfies additivity and equal treatment of equals if and only if it is a compromise rule.*

A weaker version of *equal treatment of equals* can also be formalized, stating that if two teams have the same audiences, not only when facing each of the other teams, but also when facing themselves at each stadium, then they should receive the same amount. An axiom of *pairwise reallocation proofness* (which says that a redistribution between the audiences of the two games involving a pair of teams does not affect the revenues obtained by the teams in the pair) fills the gap between both axioms (e.g., Bergantiños and Moreno-Ternero, 2022a). Thus, paralell characterization results to those listed here can be obtained upon replacing *equal treatment of equals* by the combination of *weak equal treatment of equals* and *pairwise reallocation proofness.*

We now consider three natural axioms reflecting focal bounds. The first one says that each team should receive, at most, the total audience of the games it played. The second one says that each team should receive, at most, the total audience of all games in the tournament. The third axiom says that no team should receive negative awards. Formally,

Maximum aspirations: For each $A \in \mathcal{P}$ and each $i \in N$, $R_i(A) \leq \alpha_i(A)$.

Weak upper bound: For each $A \in \mathcal{P}$ and each $i \in N$, $R_i(A) \leq ||A||$.

Non-negativity: For each $A \in \mathcal{P}$ and each $i \in N$, $R_i(A) \geq 0$.

The three axioms, together with *additivity* and *equal treatment of equals,* shrink the family of compromise rules in meaningful ways.

Theorem 3 *(Bergantiños and Moreno-Ternero, 2021, 2022a) The following statements hold:*

1. *A rule satisfies additivity, equal treatment of equals and maximum aspirations if and only if it is a compromise rule where* $\lambda \in \left[\frac{n-2}{2(n-1)}, 1\right]$.

2. *A rule satisfies additivity, equal treatment of equals and non-negativity if and only if it is a compromise rule where* $\lambda \in \left[\frac{-1}{n-1}, \frac{n-2}{2(n-1)}\right]$.

3. *A rule satisfies additivity, equal treatment of equals and weak upper bound if and only if it is a compromise rule where* $\lambda \in \left[1 - \frac{n}{2}, 1\right]$.

[5]Impartiality has a long tradition in the theory of justice (e.g., Moreno-Ternero and Roemer, 2006).

If *equal treatment of equals* is replaced by *anonymity* in Theorem 3.1, a larger family of rules is characterized (e.g., Bergantiños and Moreno-Ternero, 2023c). Those rules extend the compromise rules by means of linearly combining it with the outcome of applying *concede-and-divide* to an auxiliary problem with some mullified audiences. Specific subsets of these rules are characterized adding axioms of *order preservation* (e.g., if the home/away audience of team i is, game by game, not smaller than the audience of team j, then that team i should not receive less than team j) to those in Theorem 3 (e.g., Bergantiños and Moreno-Ternero, 2023c).

An important member of the compromise rules is the one obtained for the parameter $\lambda = \frac{n-2}{2(n-1)}$. That is,

Equal-split rule, ES: for each $A \in \mathcal{P}$, and each $i \in N$,

$$ES_i(A) = \frac{\alpha_i}{2}.$$

As described in Bergantiños and Moreno-Ternero (2020a), viewers of each game can essentially be divided in two categories: *fans* and *no fans*. As the name suggests, the former are those watching the game because they are fans of one of the teams playing. The latter are those watching the game because they thought that the specific combination of teams rendered the game interesting. It is natural to assume that the revenue generated by *fans* should be allocated to the corresponding team, whereas the revenue generated by the *no fans* should be divided equally between both teams. The *equal-split* rule and *concede-and-divide* are two extreme rules from the point of view of treating fans. The former assumes that only *no fans* exist. The latter assumes that there are as many fans as possible (compatible with the real data). The following two basic axioms allow to distinguish both rules further axiomatically.

Null team: For each $A \in \mathcal{P}$, and each $i \in N$ such that $a_{ij} = 0 = a_{ji}$, for all $j \in N$, $R_i(A) = 0$.

Essential team: For each $A \in \mathcal{P}$, and each $i \in N$ such that $a_{jk} = 0$ for all $\{j,k\} \in N \backslash \{i\}$, $R_i(A) = \alpha_i(A)$.

Theorem 4 *(Bergantiños and Moreno-Ternero, 2020a) The following statements hold:*

1. *A rule satisfies additivity, equal treatment of equals, and null team if and only if it is the equal-split rule.*
2. *A rule satisfies additivity, equal treatment of equals or anonymity, and essential team if and only if it is concede-and-divide.*

Note that the second statement in the previous result allows to interchange the axioms of *equal treatment of equals* and *anonymity*. This is not the case with the first statement. To wit, for each $\lambda \in \mathbb{R}$ and each game (i,j), S^λ divides the audience a_{ij} among the teams i and j proportionally to $(1-\lambda, \lambda)$. Formally, for each $A \in \mathcal{P}$ and each $i \in N$,

$$S_i^\lambda(A) = \sum_{j \in N \backslash \{i\}} (1-\lambda) a_{ij} + \sum_{j \in N \backslash \{i\}} \lambda a_{ji}.$$

The *equal-split rule* corresponds to the case where $\lambda = 0.5$. When $\lambda = 0$ all the audience is assigned to the home team and when $\lambda = 1$ all the audience is assigned to the away team. We say that R is a **split rule** if $R \in \left\{S^\lambda : \lambda \in [0,1]\right\}$. We say that R is a **generalized split rule** if $R \in \left\{S^\lambda : \lambda \in \mathbb{R}\right\}$.

Theorem 5 *(Bergantiños and Moreno-Ternero, 2023c) The following statements hold:*

1. *A rule satisfies additivity, anonymity, and null team if and only if it is a generalized split rule.*
2. *A rule satisfies additivity, anonymity, null team and either maximum aspirations, weak upper bound or non-negativity if and only if it is a split rule.*

The *split rules* are also characterized when the following *monotonicity* axiom is considered.[6]

Team monotonicity. For each pair $A, A' \in \mathcal{P}$ and each $i \in N$,

$$\left.\begin{array}{l} a_{ij} \leq a'_{ij} \text{ for each } j \in N \setminus \{i\} \text{ and} \\ a_{ji} \leq a'_{ji} \text{ for each } j \in N \setminus \{i\} \end{array}\right\} \quad \Rightarrow \quad R_i(A) \leq R_i(A').$$

Theorem 6 *(Bergantiños and Moreno-Ternero, 2022c) A rule satisfies weak equal treatment of equals and team monotonicity if and only if it is a split rule.*

All the results presented above make use of *additivity.* The following alternative axioms formalizing the notion of *marginalism* have also been considered, giving rise to other characterizations.

The next axiom states that the if we have additional viewers in a game, then the involved teams (respectively the non-involved teams) should be affected in the same amount. Formally,

Equal benefits from additional viewers: For each pair A, $A' \in \mathcal{P}$ such that $a_{ij} = a'_{ij}$, for each pair $(i,j) \neq (i_0, j_0)$, and $a_{i_0,j_0} < a'_{i_0,j_0}$, we have

$$R_{i_0}(A') - R_{i_0}(A) = R_{j_0}(A') - R_{j_0}(A),$$

and

$$R_i(A') - R_i(A) = R_j(A') - R_j(A),$$

when $\{i,j\} \subset N \setminus \{i_0, j_0\}$.

We now introduce a group of axioms that state how a rule should react when additional viewers (of some specific team) appear. More precisely, let A, $A' \in \mathcal{P}$ and $i \in N$ such that $a_{ij} \leq a'_{ij}$ and $a_{ji} \leq a'_{ji}$ for each $j \in N \setminus \{i\}$ and $a_{jk} = a'_{jk}$ when $i \notin \{j,k\}$. How should a rule manage those extra viewers? We consider three possible ways.

First, we ignore that all viewers come from games involving team i and assume that all teams should equally share those additional viewers. Formally,

Equal sharing of additional team viewers: For each pair A, $A' \in \mathcal{P}$, and each $i \in N$ such that $a_{ij} \leq a'_{ij}$ and $a_{ji} \leq a'_{ji}$ for each $j \in N \setminus \{i\}$ and $a_{jk} = a'_{jk}$ when $i \notin \{j,k\}$, then there exists $c \in \mathbb{R}$ such that for each $l \in N$,

$$R_l(A') - R_l(A) = c.$$

[6] *Monotonicity* axioms have a long tradition in axiomatic work that can be traced back to Thomson and Myerson (1980), among others. Alcantud et al., (2022) is a recent instance. Bergantiños and Moreno-Ternero (2022b, 2022c) explore alternative *monotonicity* axioms and their implications in this setting of broadcasting problems.

Second, as the audience of team i has increased the same amount than the audience of the rest of the teams (combined), team i should increase as much as the rest of the teams combined. Formally,

Half sharing of additional team viewers: For each pair A, $A' \in \mathcal{P}$, and each $i \in N$ such that $a_{ij} \leq a'_{ij}$ and $a_{ji} \leq a'_{ji}$ for each $j \in N \setminus \{i\}$ and $a_{jk} = a'_{jk}$ when $i \notin \{j,k\}$, then

$$R_i(A') - R_i(A) = \sum_{l \in N \setminus \{i\}} (R_l(A') - R_l(A)).$$

Third, we assume that team i should be awarded with all the revenue generated by those viewers. Formally,

No sharing of additional team viewers: For each pair A, $A' \in \mathcal{P}$, and each $i \in N$ such that $a_{ij} \leq a'_{ij}$ and $a_{ji} \leq a'_{ji}$ for each $j \in N \setminus \{i\}$ and $a_{jk} = a'_{jk}$ when $i \notin \{j,k\}$, then

$$R_i(A') - R_i(A) = ||A'|| - ||A||.$$

Theorem 7 *(Bergantiños and Moreno-Ternero, 2020b) The following statements hold:*

1. *A rule satisfies equal benefits from additional viewers and null team if and only if it is the equal-split rule.*
2. *A rule satisfies equal benefits from additional viewers and essential team if and only if it is concede-and-divide.*
3. *A rule satisfies equal treatment of equals and equal sharing of additional team viewers if and only if it is the uniform rule.*
4. *A rule satisfies equal treatment of equals and half sharing of additional team viewers if and only if it is the equal-split rule.*
5. *A rule satisfies equal treatment of equals and no sharing of additional team viewers if and only if it is concede-and-divide.*

We conclude this section describing some open questions within the axiomatic approach.

On the one hand, the three axioms just described formalizing the impact of additional team viewers say explicitly how tho share this additional revenue. One could think of less demanding axioms giving some freedom to formalize the way in which the additional revenue is shared. Is it possible to characterize some interesting rules with such axioms?

On the other hand, all the characterizations mentioned above assume that the population (N) is fixed. It seems natural to obtain characterizations for a variable-population setting. Two classical variable-population axioms are *consistency* and *population monotonicity* (e.g. Thomson, 2011). The former states that when for each problem and for the "reduced problem" obtained by imagining the departure of a group of agents with their allocation, and reassessing the remaining amount to the remaining agents, it chooses the allocation proposed by the rule to this subgroup. The latter states that if a new team joins a league, then no team from the initial league worsen. It would be interesting to explore the implications of those axioms.

3 A game-theoretical approach

A natural course of action in economics is to solve problems indirectly, via associating them to cooperative games for which we have well-established solutions. Formally, a **cooperative game with transferable utility**, briefly a **TU game**, is a pair (N, v), where N denotes a set of agents and $v : 2^N \to \mathbb{R}$ satisfies $v(\varnothing) = 0$. As the population N will remain fixed, we avoid its use in the notation.

The **core** is defined as the set of feasible payoff vectors, upon which no coalition can improve. Formally,

$$Core\,(v) = \left\{ x \in \mathbb{R}^N \text{ such that } \sum_{i \in N} x_i = v\,(N) \text{ and } \sum_{i \in S} x_i \geq v\,(S)\,, \text{ for each } S \subset N \right\}.$$

The **Shapley value** (Shapley, 1953) is defined for each player as the average of his contributions across orders of agents. Formally, for each $i \in N$,

$$Sh_i\,(v) = \frac{1}{n!} \sum_{\pi \in \Pi_N} \left[v\,(Pre\,(i, \pi) \cup \{i\}) - v\,(Pre\,(i, \pi)) \right],$$

where Π_N denotes the set of all orders on N, and $Pre\,(i, \pi) = \{ j \in N \mid \pi\,(j) < \pi\,(i) \}$.

The **egalitarian value** (e.g., van den Brink, 2007) yields each agent an equal portion of the value of the grand coalition. Formally, for each $i \in N$,

$$ED_i\,(v) = \frac{1}{n} v(N),$$

The **egalitarian Shapley values** (e.g., Casajus and Huettner, 2013, van den Brink et al., 2013, and Casajus and Yokote, 2019) are obtained with the convex combinations of the previous values. Formally, for each $i \in N$,

$$S_i^{\lambda}\,(v) = \lambda Sh_i\,(v) + (1 - \lambda) ED_i\,(v)\,,$$

Bergantiños and Moreno-Ternero (2020a) associate with each broadcasting problem $A \in \mathcal{P}$ a TU game v_A. To do so, they take an optimistic stance on what revenue a coalition can generate on its own. To wit, the highest possible revenue that a game between teams i and j in the former's stadium may generate is a_{ij}. Thus, by breaking away from the league, the most optimistic scenario for any coalition of teams is to generate the same revenue they generated before leaving the league. Formally, for each $S \subset N$, $v_A\,(S)$ is defined as the total audience of the games played by the teams in S. Namely,

$$v_A\,(S) = \sum_{i,j \in S, i \neq j} a_{ij} = \sum_{i,j \in S, i < j} (a_{ij} + a_{ji})\,.$$

Bergantiños and Moreno-Ternero (2020a) show the correspondence between the *equal-split rule* and the Shapley value of the corresponding TU game via the previous association. Bergantiños and Moreno-Ternero (2022a) argue that there is also a correspondence between the *uniform rule* and the equal division value of the corresponding TU game. Consequently, there is also a correspondence between the rules that compromise between the *uniform rule* and the *equal-split rule* (that is, the rules that belong to the *compromise rules* for $\lambda \in \left[0, \frac{n-2}{2(n-1)}\right]$) and the egalitarian Shapley values of the corresponding TU game. Gonçalves-Dosantos *et al* (2022) have recently introduced a new value in cooperative games. This value is characterized with an axiom called *necessary players*, which is reminiscent of the *essential team* axiom introduced above. It remains an open question to explore whether there is a connection between *concede-and-divide* and such a value (or related ones). Similarly, the connections between other families of rules in broadcasting problems and families of values in cooperative game have not been explored yet.

Bergantiños and Moreno-Ternero (2020a) also show that, in order to satisfy the core constraints (of the game v_A), we should divide the revenue generated by the audience of a game between the two teams playing the game. Formally,

Theorem 8 *(Bergantiños and Moreno-Ternero, 2020a) Let* $A \in \mathcal{P}$ *and* v_A *be its associated TU game. Then,* $x = (x_i)_{i \in N} \in Core\,(v_A)$ *if and only if, for each* $i \in N$*, there exist* $\left(x_i^j\right)_{j \in N \setminus \{i\}}$ *satisfying three conditions:*
(i) $x_i^j \geq 0$*, for each* $j \in N \setminus \{i\}$*;*
(ii) $\sum_{j \in N \setminus \{i\}} x_i^j = x_i$*;*
(iii) $x_i^j + x_j^i = a_{ij} + a_{ji}$*, for each* $j \in N \setminus \{i\}$*.*

The game we have described in this section is formally equivalent to the game associated by van den Nouweland et al., (1996) to the so-called Terrestial Flight Telephone System. They prove that such a game is convex and, therefore, its Shapley value belongs to the core (e.g., Shapley, 1953). Thus, it would also follow from there that the *equal-split rule* always satisfies core selection (which can also be inferred from Theorem 8). Van den Nouweland et al., (1996) also show that, in this game, the Shapley value coincides with the Nucleolus and the τ-value. Thus, all these values collapse into the *equal-split rule*, which reinforces this rule from a game-theoretical approach.

Other plausible cooperative games could also be associated to broadcasting problems (for instance, taking a more pesimistic stance on what revenue a coalition can generate on its own). It remains an open question to study such alternative games and their connections to broadcasting problems. Alternatively, one could also consider to solve broadcasting problems indirectly too, but via associating *claims problem* to them, instead of cooperative games.

O'Neill (1982) introduced the so-called *claims problem* where an amount of a perfectly divisible good (the endowment) has to be allocated among a group of agents who hold claims against it, and the aggregate claim is higher than the endowment. See Thomson (2003, 2015a, 2019a) for excellent surveys of this literature. Formally, a *claims problem* is a triple consisting of a population N, a claims profile $c \in \mathbb{R}_+^n$, and an *endowment* $E \in \mathbb{R}_+$ such that $\sum_{i \in N} c_i \geq E$. Let $C \equiv \sum_{i \in N} c_i$. Given a claims problem $(N, c, E) \in \mathcal{D}$, an *allocation* is a vector $x \in \mathbb{R}^n$ satisfying that, for each $i \in N$, $0 \leq x_i \leq c_i$ and $\sum_{i \in N} x_i = E$. Let $\mathcal{D}$ be the domain of claims problems so defined. A *rule* on $\mathcal{D}$, $R \colon \mathcal{D} \to \mathbb{R}^n$, associates with each problem $(N, c, E) \in \mathcal{D}$ an allocation $R\,(N, c, E)$ for the problem. Two instances are the following:

The **proportional** rule, which yields awards proportionally to claims. Formally, for each $(N, c, E) \in \mathcal{D}$,

$$P(N, c, E) = \frac{E}{C}\, c.$$

The **Talmud** rule, which equalizes awards or losses depending on whether the endowment is above or below one half of the aggregate claim, and using half-claims instead of claims. Formally, for each $(N, c, E) \in \mathcal{D}$, it selects

$$T_i(N,c,E) = \begin{cases} \min\left\{\frac{c_i}{2},\lambda\right\} & \text{if } E \leq \frac{1}{2}C \\ \max\left\{\frac{c_i}{2}, c_i - \mu\right\} & \text{if } E \geq \frac{1}{2}C \end{cases}$$

where λ and μ are chosen so that $\sum_{i\in N} T_i(N,c,E) = E$.

Bergantiños and Moreno-Ternero (2020a) associate with each $A \in \mathcal{P}$ a claims problem $\left(N, c^A, E^A\right) \in \mathcal{D}$ where $c_i^A = \alpha_i$, for each $i \in N$, and $E^A = ||A||$. They also show that there is a correspondence between the *equal-split rule* and the proportional and Talmud rules for the associated claims problem. Other prominent rules exist within the literature of claims problems (for instance, the so-called constrained equal awards and constrained equal losses rules). It remains an open question to explore the allocations both rules would produce for broadcasting problems. Somewhat related, it would be interesting to explore the possible connections between the several families of rules for broadcasting problems mentioned above and the several families of rules that have been considered in the literature on claims problems (e.g., Moreno-Ternero and Villar, 2006; van den Brink and Moreno-Ternero, 2017; Thomson, 2019).

We conclude this section connecting the two indirect approaches presented in it. To wit, it is possible to apply the cooperative game theory approach to the conflicting claims problem. Thus, the claims problems associated to a broadcasting problem A can be mapped into a TU game w_A known as the bankruptcy game (e.g., Aumann and Maschler, 1985). For each $A \in \mathcal{P}$, and for each $S \subset N$,

$$w_A(S) = \max\left\{\sum_{i,j\in S} a_{ij} - \sum_{i,j\notin S} a_{ij}, 0\right\}.$$

When $n = 3$, this game coincides with the one introduced above, i.e., $v_A = w_A$. Nevertheless, when $n > 3$, v_A and w_A could be different and it remains an open question to explore the latter.

4 A statistical estimation approach

Bergantiños and Moreno-Ternero (2020a) also take a different approach, based on a form of statistical estimation. In general, it is assumed that individuals watching a game involving teams i and j can be classified in four buckets. First, being a fan of this sport per se. Second, being a fan of team i (in which case one would watch all the games involving team i). Third, being a fan of team j (in which case one would watch all the games involving team j). Fourth, being a fan of the game between teams i and j. In practice, the above information is not available and we only know the total audience of the game. Thus, we will try to estimate the number of fans in each category.

Formally, for each pair of teams $i, j \in N$, with $i \neq j$, let

$$a_{ij} = b_0 + b_i + b_j + \varepsilon_{ij},$$

where b_0 denotes the number of generic sport fans, b_k denotes the number of fans of team $k = i, j$, and ε_{ij} denotes the number of joint fans for the pair $\{i, j\}$.

Fix $k \in N$, and consider the following minimization problem:

$$\min_{b \in \mathbb{R}^n} \sum_{i,j \in N, i \neq j} \varepsilon_{ij}^2 \tag{1}$$

where

$$\varepsilon_{ij} = \begin{cases} a_{ij} - b_0 - b_i - b_j & \text{if } k \notin \{i,j\} \\ a_{ij} - b_0 - b_i & \text{if } k = j \\ a_{ij} - b_0 - b_j & \text{if } k = i \end{cases}$$

Let $\hat{b}_0$ and $\left\{\hat{b}_i\right\}_{i \in N \setminus \{k\}}$ denote the solutions to (1). Finally, for each pair $i, j \in N$, with $i \neq j$, let

$$\hat{\varepsilon}_{ij} = \begin{cases} a_{ij} - \hat{b}_0 - \hat{b}_i - \hat{b}_j & \text{if } k \notin \{i,j\} \\ a_{ij} - \hat{b}_0 - \hat{b}_i & \text{if } k = j \\ a_{ij} - \hat{b}_0 - \hat{b}_j & \text{if } k = i \end{cases}$$

The following procedures are then considered to allocate a_{ij}:

$(P1)$ $\hat{b}_0$ is divided equally among all teams.

$(P2)$ $\hat{b}_l$ is assigned to team l, for each $l \in N \setminus \{k\}$.

$(P3)$ $\hat{\varepsilon}_{ij}$ is divided equally between teams i and j, for each pair $i, j \in N$, with $i \neq j$.

The above suggests the following rule:

$$R_i^{b,k}(A) = \begin{cases} (n-1)\,\widehat{b_0} + 2\,(n-1)\,\widehat{b_i} + \sum_{j \in N \setminus \{i\}} \frac{\widehat{\varepsilon_{ij}} + \widehat{\varepsilon_{ji}}}{2} & \text{if } i \neq k \\ (n-1)\,\widehat{b_0} + \sum_{j \in N \setminus \{i\}} \frac{\widehat{\varepsilon_{ij}} + \widehat{\varepsilon_{ji}}}{2} & \text{if } i = k \end{cases} \tag{2}$$

One might argue that the above allocation would depend on k. This is not the case. The next theorem actually states that the allocation rule, so constructed, coincides with *concede-and-divide* (hence its name).

Theorem 9 *(Bergantiños and Moreno-Ternero, 2020a) For each $A \in \mathcal{P}$ and each pair $i, k \in N$, let $R_i^{b,k}(A)$ be the allocation obtained by applying formula* (2)*. Then,*

$$R_i^{b,k}(A) = \frac{(n-1)\,\alpha_i - ||A||}{n-2} = CD_i(A).$$

As an open question from this approach, we mention that the above assumes that viewers of the game between i and j can be classified in four categories: being a fan of this sport per se (which corresponds to b_0); being a fan of team i (which corresponds to b_i); being a fan of team j (which corresponds to b_j); and those considering that the game between teams i and j is interesting (which correspond to ε_{ij}). It is plausible to consider alternative partitions of viewers. The ensuing estimation of the parameters would likely yield alternative allocation rules for broadcasting problems.

5 A decentralized approach

The previous sections gathered normative and positive foundations for many rules to share revenues raised from broadcasting. Nevertheless, no strong consensus exists over specific rules (albeit two of them; namely equal-split and concede-and-divide seem to be salient). This motivates to explore a different (decentralized) approach, following a long tradition of allocating resources by voting (e.g., Birnberg et al., 1970; Barzel and Sass, 1990), in which the choice of a rule could be made by means of simple majority voting, letting each team vote for a rule.

Given a problem $A \in \mathcal{P}$, we say that $R(A)$ is a *majority winner* (within the set of rules $\mathcal{R}$) for A if there is no other rule $R' \in \mathcal{R}$ such that $R'_i(A) > R_i(A)$ for a majority of teams. We say that the family of rules $\mathcal{R}$ has a *majority voting equilibrium* if there is at least one majority winner (within $\mathcal{R}$) for each problem $A \in \mathcal{P}$.

There is no majority voting equilibrium for the family of general rules (e.g., Bergantiños and Moreno-Ternero 2023a). The underlying rationale is that given a general rule, one can construct another general rule which, at a certain problem, increases the amount obtained by a majority of the teams involved, while reducing the amount obtained by all of the others.[7] Nevertheless, the existence of a majority voting equilibrium is guaranteed for other sufficiently large subfamilies of rules. In particular, for the family of compromise rules. This is a consequence of the fact that those rules satisfy the so-called *single-crossing* property (e.g., Bergantiños and Moreno-Ternero 2023a). That is, for each pair of rules within the family, and each problem, there exists a team separating those teams benefitting from the choice of one rule and those benefitting from the choice of the other. It is well known that the single-crossing property of preferences is a sufficient condition for the existence of a majority voting equilibrium (Gans and Smart 1996). Thus, the next result follows.

Theorem 10 *(Bergantiños and Moreno-Ternero 2023a). There is a majority voting equilibrium for each bounded family of compromise rules* $\left\{UC^{\lambda}\right\}_{\lambda\in[\underline{\lambda},\overline{\lambda}]}$.

Theorem 10 states that if we let teams vote for a rule within any bounded family of compromise rules, then there will be a majority winner for each problem. The identity of this winner will be problem specific and it will depend on the characteristics of the problem at stake. In most cases, either the *equal-split rule* or *concede-and-divide* arise.

Another consequence of the single-crossing property is that it guarantees progressivity comparisons of schedules (Jakobsson 1976; Hemming and Keen 1983). Thus, we can also obtain an interesting result, referring to the distributive power of the rules within the family of compromise rules. Formally, given $x, y \in \mathbb{R}^n$ satisfying $x_1 \leq x_2 \leq ... \leq x_n$, $y_1 \leq y_2 \leq ... \leq y_n$, and $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i$, we say that x *is greater than* y *in the Lorenz ordering* if $\sum_{i=1}^{k} x_i \geq \sum_{i=1}^{k} y_i$, for each $k = 1, ..., n-1$, with at least one strict inequality. When x is greater than y in the Lorenz ordering, one can state (see, for instance, Dasgupta et al., 1973) that x is unambiguously "more egalitarian" than y. In our setting, we say that a rule R *Lorenz dominates* another rule R' if for each $A \in \mathcal{P}$, $R(A)$ is greater than $R'(A)$ in the Lorenz ordering. As the Lorenz criterion is a partial ordering, one might not expect to be able to perform many comparisons of vectors. It turns out, however, that the compromise rules are fully ranked according to this criterion.

[7]Similar arguments have been made in related models (e.g., Marhuenda and Ortuño-Ortín, 1998; Moreno-Ternero, 2011).

Proposition 1 *(Bergantiños and Moreno-Ternero 2023a). The following statements hold:*

- *If* $0 \leq \lambda_1 \leq \lambda_2$ *then* UC^{λ_1} *Lorenz dominates* UC^{λ_2}.
- *If* $\lambda_1 \leq \lambda_2 \leq 0$ *then* UC^{λ_2} *Lorenz dominates* UC^{λ_1}.

Proposition 1 implies that the parameter defining the family can actually be interpreted as an index of the distributive power of the rules within the family. The *uniform rule* is the center of the family, obtained when $\lambda = 0$. It also happens to be the *maximal* element of the Lorenz ordering, as it generates fully egalitarian allocations. It is then obvious that all other rules within the family are Lorenz dominated by it. The remarkable feature, that Proposition 1 states, is that departing from the *uniform rule* in both directions (either with positive parameters or with negative parameters) we obtain rules that yield progressively less egalitarian allocations. That is, the more we depart from the center, the less egalitarian rules become. And we can establish those comparisons for each pair of rules within each of the two sides of the family. When the pair of rules is made of rules in different sides of the family (i.e., one corresponding to a negative parameter and the other corresponding to a positive parameter) then we cannot establish Lorenz comparisons for such a pair of rules.

Alternative forms of decentralization (via voting) could be explored. For instance, there exists a growing interest to consider *approval voting* (Brams and Fishburn 1978), as an alternative to majority voting in many instances. An alternative to *approval voting* is *cumulative voting* (Glasser, 1959; Sawyer and MacRae, 1962). An interesting case is the one in which every agent is endowed with a fixed number of votes that are evenly divided among all candidates for whom she votes. This corresponds to the notion of *Shapley ranking* introduced by Ginsburgh and Zang (2003) for the so-called museum pass game, which can be rationalized as the Shapley value of an associated cooperative game with transferable utility.[8] As we mentioned in the game-theoretical approach for broadcasting problems, the Shapley value is naturally associated to the *equal-split rule*. It seems plausible to conjecture that such a rule would arise as the equilibrium in a decentralized process with *Shapley ranking* as an alternative to majority voting.

6 Cancelled seasons and the operational approach

We conclude the theoretical part of this survey exploring extensions of the model presented above. We start with a natural option, introduced in Bergantiños and Moreno-Ternero (2023d) to accommodate the case where a league has been cancelled. This was particularly relevant in the aftermath of the COVID-19 pandemic, which forced the partial or total cancellation of many sports competitions worldwide. The question that arises is how the allocation of the broadcasting revenues should be modified.

In the general setting, a **problem** is defined as a pair (A, E), where $A \in \mathcal{A}_{n\times n}$ is a matrix and $E \in \mathbb{R}_+$ is an endowment to be allocated among teams in N, based on the audience matrix. We write $a_{ij} = \varnothing$ if the game was cancelled. Notice that the problems corresponding to fully completed seasons, defined as above, correspond with the case where $a_{ij} \neq \varnothing$, for each pair $i, j \in N$, with $i \neq j$ and $E = ||A||$.

[8] See also Ginsburgh and Moreno-Ternero (2023).

We now define an **extension operator** via a mapping assigning to each problem with possible empty entries in the audience matrix a benchmark problem without any empty entries, with the proviso that non-empty entries in the original matrix of audiences remain unchanged.[9] Two extension operators arise naturally. First, the one associating to a cancelled game a zero audience. Second, the one associating to a cancelled game the audience of the game in the first leg of the tournament, or zero if such a game was also cancelled. Formally,

Zero, z: For each pair $i, j \in N$,

$$a_{ij}^{z} = \begin{cases} 0 & \text{if } a_{ij} = \varnothing \\ a_{ij} & \text{if } a_{ij} \neq \varnothing. \end{cases}$$

Leg, ℓ: For each pair $i, j \in N$,

$$a_{ij}^{\ell} = \begin{cases} a_{ji} & \text{if } a_{ij} = \varnothing \text{ and } a_{ji} \neq \varnothing \\ 0 & \text{if } a_{ij} = \varnothing \text{ and } a_{ji} = \varnothing \\ a_{ij} & \text{if } a_{ij} \neq \varnothing. \end{cases}$$

For each operator o, and each benchmark rule R, we can define an extended rule R^{o} in the obvious way. Some instances are the **zero-extended equal-split rule** (ES^{z}), the **zero-extended concede-and-divide** (CD^{z}), the **leg-extended equal-split rule** (ES^{ℓ}), and the **leg-extended concede-and-divide** (CD^{ℓ}).

The next two axioms are natural extensions of two axioms defined above.

Null team on non-cancelled games: For each problem (A, E) with $||A|| > 0$ and each $i \in N$, such that for each $j \in N \backslash \{i\}$, $a_{ij} \in \{0, \varnothing\}$ and $a_{ji} \in \{0, \varnothing\}$, $R_i(A, E) = 0$.

Essential team on non-cancelled games: For each problem (A, E), and each $i \in N$ such that $a_{jk} \in \{0, \varnothing\}$ for each pair $\{j, k\} \subset N \backslash \{i\}$,$R_i(A, E) = E$.

Baseline monotonicity compares the allocation in two problems obtained by modifying the audience of the game played by i and j at i's stadium. If the audience has increased, baseline monotonicity says that teams i and j could not receive less whereas the rest of the teams could not receive more, assuming that the total revenue is the same. If the game has only been played at the second problem, then we apply the same idea, but comparing the audience of that game with the audience given by the operator to the cancelled game in the first problem.

O-baseline monotonicity: Let o be an operator and two problems $(A, E), (A', E)$ for which there exist $i, j \in N$ such that $a'_{ij} \neq \varnothing$ and $a'_{kl} = a_{kl}$ for each $(k, l) \neq (i, j)$. Then, two conditions hold:

1. For each $k \in \{i, j\}$,

$$\begin{aligned} R_k(A', E) &\geq R_k(A, E) \text{ when } a'_{ij} \geq a_{ij}^{o}, \\ R_k(A', E) &\leq R_k(A, E) \text{ when } a'_{ij} \leq a_{ij}^{o}. \end{aligned}$$

[9]The concept of operators on the space of allocation rules is explored in detail by Thomson and Yeh (2008) and Thomson (2019). See also Hougaard et al. (2012) and Moreno-Ternero and Vidal-Puga (2021).

2. For each $k \in N \setminus \{i, j\}$,

$$R_k(A', E) \leq R_k(A, E) \text{ when } a'_{ij} \geq a^o_{ij},$$
$$R_k(A', E) \geq R_k(A, E) \text{ when } a'_{ij} \leq a^o_{ij}.$$

Reallocation proofness compares two problems where the aggregate audience of a given team, as well as the aggregate audience, coincide. The axiom says that this team should receive the same in both problems.

Reallocation proofness: Let $(A, E), (A', E)$ and $i \in N$ be such that $\alpha_i(A) = \alpha_i(A')$ and $||A|| = ||A'||$. Then, $R_i(A, E) = R_i(A', E)$.

Weak reallocation proofness is defined by requiring *reallocation proofness* only for tournaments in which no game has been cancelled.

In the last axioms we assume leagues are divided into conferences. Suppose that only games among teams in the same conference have a positive audience. Then, instead of solving the whole problem, we can solve each conference problem separately, assuming that the revenue is divided among the conference problems proportionally to their audiences, computed through the operator. We consider two axioms, depending on how we define the conference problem. In the single-conference axiom, each team plays a single problem (the one given by the teams of its conference). In the multi-conference axiom, each team plays several problems. For each conference, we consider a problem in which all teams participate, but only the games involving the teams within the conference have been played.

Formally, for each $(A, E) \in \mathcal{P}$, and each $S \subset N$, we consider two ways of modeling the tournament induced by A among teams in S. Let $A^S \in \mathcal{A}_{|S| \times |S|}$ be such that $a^S_{ij} = a_{ij}$ for all $i, j \in S$. Besides, let $A^{S,\varnothing} \in \mathcal{A}_{n \times n}$ be such that $a^{S,\varnothing}_{ij} = a_{ij}$ when $i, j \in S$ and $a^{S,\varnothing}_{ij} = \varnothing$ otherwise. Notice that in A^S the set of teams is S whereas in $A^{S,\varnothing}$ the set of teams is N.

We denote by $||A(S)||$ the aggregate audience of all games played among teams within S. Namely,

$$||A(S)|| = \sum_{i,j \in S, a_{ij} \neq \varnothing} a_{ij}.$$

We say that $\{N_1, ..., N_p\}$ is a partition of N if $N = \bigcup_{k=1}^{p} N_k$, $N_k \cap N_{k'} = \varnothing$ for each pair $k \neq k'$, and $N_k \neq \varnothing$ for each $k = 1, ..., p$.

O-single-conference (SC^o): Let (A, E), $\{N_1, ..., N_p\}$ a partition of N such that if $a_{ij} > 0$ with $i \in N_{i'}$ and $j \in N_{j'}$ then $i' = j'$. For each $i \in N_{i'}$,

$$R_i(A, E) = R_i\left(A^{N_{i'}}, \frac{||A^o(N_{i'})||}{\sum_{k=1}^{p} ||A^o(N_k)||} E\right).$$

O-multi-conference (MC^o): Let (A, E), $\{N_1, ..., N_p\}$ a partition of N such that if $a_{ij} > 0$ with $i \in N_{i'}$ and $j \in N_{j'}$ then $i' = j'$. For each $i \in N$,

$$R_i(A, E) = \sum_{k=1}^{p} R_i\left(A^{N_k,\varnothing}, \frac{||A^o(N_k)||}{\sum_{k=1}^{p} ||A^o(N_k)||} E\right).$$

We now present the characterization results.

Theorem 11 *(Bergantiños and Moreno-Ternero, 2023d) The following statements hold:*

1. *A rule satisfies reallocation proofness and single-conference if and only if it is zero-extended equal-split rule.*

2. *A rule satisfies reallocation proofness, null team for non-cancelled games and multi-conference if and only if it is zero-extended equal-split rule.*

3. *A rule satisfies weak reallocation proofness, single-conference, and leg-baseline monotonicity if and only if it is leg-extended equal-split rule.*

4. *A rule satisfies weak reallocation proofness, leg-baseline monotonicity, null team for non-cancelled games and multi-conference if and only if it is leg-extended equal-split rule.*

5. *A rule satisfies reallocation proofness, essential team on non-cancelled games, and multi-conference if and only if it is zero-extended concede-and-divide.*

6. *A rule satisfies weak reallocation proofness, essential team on non-cancelled games, multi-conference, and leg-baseline monotonicity if and only if it is leg-extended concede-and-divide.*

We end this section by mentioning some open questions within the operational approach.

A **basic operator** is a mapping from the set of rules onto itself. Formally, a mapping $O : \mathcal{R} \to \mathcal{R}$. For instance, the one associating to a rule its dual from the claims vector. Formally,

α-dual, O^{α}: For each $R \in \mathcal{R}$, and each $A \in \mathcal{P}$

$$O^{\alpha}(R)(A) = \alpha - R(A)$$

A natural question is whether operators preserve properties. That is, if a given rule R satisfies a certain axiom, is it also the case that $O(R)$, the image of that rule via the operator, satisfies the same axiom. If so, we say the operator preserves the axiom. The dual operator presented above preserves some axioms, but not others. A systematic study, which would uncover the structure of the problem further, is an open item for further research.

More generally, we can consider operators that associate to a pair of rules a new one. Formally, a mapping $O : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$.

Bi-operators, O^{λ}: For each $\lambda \in \mathbb{R}$, each pair $R, S \in \mathcal{R}$, and each $A \in \mathcal{P}$,

$$O^{\lambda}(R, S)(A) = R(A) + \lambda(R(A) - S(A)).$$

For instance $O^{\frac{2}{n-2}}(U, ES) \equiv UC^{\{\frac{-1}{n-1}\}}$ and $O^{\frac{n}{n-2}}(ES, U) \equiv CD$.

In the case of bi-operators, the issue of preservation is naturally adapted. If the two source rules satisfy a given axiom, is it the case that the image (of those two rules) satisfies the axiom too? Again, a systematic analysis of this issue is pending as an open question for further research.

7 Empirical illustrations

Although the contents presented in the previous sections are of a theoretical nature, they can obviously be applied to real-life cases. Bergantiños and Moreno-Ternero (2020a, 2021, 2023b) present empirical applications resorting to data from La Liga, the Spanish Football League.[10]

La Liga is a standard round robin tournament involving 20 teams. Thus, each team plays 38 games, facing each time one of the other 19 teams (once home, another away).

Bergantiños and Moreno-Ternero (2020a) compare the allocation implemented in the season 2016-17 with the ones obtained by applying the equal-split rule or concede-and-divide. The results are presented in Table 1 below.

Insert Table 1 about here

Several conclusions can be derived from Table 1. Contrary to what some people argue, the allocation used by La Liga seems to be biased against the two powerhouses (Barcelona and Real Madrid). Although the *equal-split rule* would recommend a somewhat similar aggregated allocation for them (close to one fourth of the pie), *concede-and-divide* would recommend for them almost two fifths of the pie. Eight teams are favored by the actual allocation, in the sense that the amount they get is above the amounts suggested by the two rules. Seven teams obtain amounts between those suggested by the two rules. Five teams obtain amounts below those suggested by the two rules.

Bergantiños and Moreno-Ternero (2021) study the subset of compromise rules given by the convex combination of the *equal-split* rule and *concede-and-divide.* Namely, $\lambda ES + (1-\lambda)\, CD$ where $\lambda \in [0,1]$. In general, individuals watching a game can be classified as fans of one of the teams involved in the game, or as *neutral* viewers. In practice, the above information is not available and we only know the total audience of the game. Thus, λ can be considered as an estimation of the percentage of *neutral* viewers. Similarly, $1-\lambda$ can be considered as an estimation of the percentage of viewers who watch a game because they are fans of one of the teams playing the game. It is argued that the amount received by each team should be between the allocations proposed by the *equal-split* rule and *concede-and-divide.*

Table 2 shows the allocation put in practice for the season 2017-18 and the ones proposed by the two rules. In the last column it is checked whether the amount obtained by each team in the allocation used in practice corresponds to some compromise rule. For instance, Barcelona receives the amount that the rule $0.98ES + 0.02CD$ (namely, $\lambda = 0.98$) would yield for this setting. In contrast, Real Madrid receives less than the amount proposed by any rule within the family ($148 < \min\{158.43, 260.81\}$) whereas Atlético de Madrid receives more ($110.60 > \max\{85.77, 107.43\}$).

Insert Table 2 about here

[10]http://www.laliga.es/en

Nine teams are favored by the actual allocation, namely, the amount each gets is above the amounts suggested by both rules. Real Madrid and Betis obtain amounts below those two rules. The remaining nine teams obtain amounts between both rules. However, the parameter λ would be different for each team. For instance, for Celta, it would be the rule corresponding to $\lambda = 0.02$ (something quite similar to the *concede-and-divide* outcome), whereas, for Barcelona, it would be the rule corresponding to $\lambda = 0.98$ (something quite similar to the *equal-split* outcome).

Bergantiños and Moreno-Ternero (2023b) also study the allocation of revenues for La Liga, which is strongly regulated by the Spanish government since 2015. More precisely, the Royal Decree decomposes E (the revenue to be allocated) in four parts, each reflecting a different dimension. The amount received by each club is the sum of the amounts received in each dimension. The four dimensions and the alternatives considered are described next:

1. **Lower bounds**. Half of the total endowment is devoted to this (first) dimension. It is divided equally among all clubs, hence guaranteeing a specific *lower bound* to each: $\frac{E}{40}$.

   Two other lower bounds, defined through the literature on claims problems (e.g., O'Neill, 1982; Thomson, 2003, 2015a) are proposed.

2. **Sport performance**. One quarter of the total endowment is devoted to this (second) dimension. It is divided among clubs taking into account the sport performance during last 5 seasons.

   Two alternatives are considered. The first one is the same as in the Premier League. In the second one, each team would get, each season, a score equal to the points obtained.

3. **Economic performance**. One twelfth of the total endowment is devoted to this (third) dimension. It is divided among clubs proportionally to ticket sales in the last five seasons.

   As in the first dimension, a claims problem is associated. The amount of ticket sales is the claim of each team. Thus, four classical rules for claims problems are considered. The proportional rule is precisely the allocation implemented by La Liga. The other three are the so-called constrained equal awards, constrained equal losses, and Talmud rules.

4. **Broadcasting performance**. One sixth of the total endowment is devoted to this (last) dimension. The Royal Decree does not specify the way in which this amount should be divided among clubs. La Liga decides the amount received by each club, but it does not specify how such amounts are computed.

   Our proposals are based on the equal-split rule and concede-and-divide, respectively.

In Table 3, for the season 2017-18, it is compared the allocation of La Liga with other three allocations obtained by combining allocations of the four dimensions, as follows. In Column 3 (Low SD), for each dimension the allocation with the lowest standard deviation is selected. In Column 4 (High SD), for each dimension the allocation with the highest standard deviation is selected. In Column 5 (Average), the average of the allocations considered in such dimension is selected. For each of the three columns, the complete allocation is the sum over the allocations in each dimension.

Insert Table 3 about here

As the allocations with the lowest SD are more similar to the average, we should expect that big teams obtain more with the allocation High SD whereas small clubs obtain more with Low SD. The first four teams of the list obtain more with High SD whereas the rest obtain more with Low SD.

As we can see, no club obtains more with the average than with the maximum between Low SD and High SD. Nevertheless, Athletic Bilbao obtains more with La Liga than with the maximum between Low SD and High SD. This fact is remarkable because Low SD and High SD are some kind of extreme allocations.

In column 6 of Table 3 (Difference) we compute the difference between La Liga and the Average column. The club more favored by La Liga is Atlético Madrid obtaining 8.58 more with La Liga. The worst treated club (by far) is Betis, receiving 18.44 less.

We conclude stressing that our empirical analyses are based on La Liga. It would be interesting to perform similar analyses for other important football leagues in Europe (such as the dominant English Premier League). It would also be interesting to perform a similar analysis for some professional leagues in the US, where the broadcasting rights are divided, basically, via equal sharing (e.g., Fort and Quirk 1995).

8 Conclusion

Sports account for a large portion of all broadcasting attention. Although times are changing and new generations shift towards other forms of entertainment, sportscasting continues to be a major aspect of the entertainment industry. Massive amounts of people consume sports via broadcasting worldwide. And payments for the rights to broadcast live sports competitions have grown drastically over recent years, to the extent that they have shaped the role of sport rights in the broadcast industry (e.g., Cave and Crandall, 2001).

We have reviewed in this survey the literature on the economics of sharing the revenues from sportscast. We have concentrated on the focal case of professional sports leagues (with a double round-robin format as the benchmark case, although others could also be accommodated) in which revenues are raised collectively. We have mostly reviewed the literature on the axiomatic approach to this problem, but we have also paid attention to alternative approaches allowing to solve the problems indirectly (via associating a cooperative game or a claims problem to them, as well as taking a statistical estimation approach, or a decentralized approach via majority voting). We have also explored extensions (via the operational approach) to more general cases (for instance, those arising after leagues are cancelled) and we have illustrated the analyses to the special case of the Spanish Football League. We have also listed some open questions for further research that arise in each of those approaches. Altogether, we can safely argue that the economics of sharing the revenues from sportscast is a lively research topic nowadays.

References

[1] Adams, W.J., Yellen, J.L., 1973, Commodity Bundling and the Burden of Monopoly. Quarterly Journal of Economics 90, 475-98.

[2] Alcantud, J., Mariotti, M. Veneziani, R. 2022, Sufficientarianism, Theoretical Economics 17, 1529-1557.

[3] Aumann R.J., Maschler M., 1985. Game theoretic analysis of a bankruptcy problem from the Talmud. Journal of Economic Theory 36, 195-213.

[4] Barzel, Y., Sass, T.R., 1990. The allocation of resources by voting. Quarterly Journal of Economics 105, 745-771.

[5] Bergantiños, G., Moreno-Ternero, J.D., 2015. The axiomatic approach to the problem of sharing the revenue from museum passes. Games and Economic Behavior 89, 78-92.

[6] Bergantiños, G., Moreno-Ternero, J.D., 2020a. Sharing the revenues from broadcasting sport events. Management Science 66, 2417-2431.

[7] Bergantiños, G., Moreno-Ternero, J.D., 2020b. Allocating extra revenues from broadcasting sports leagues. Journal of Mathematical Economics 90, 65-73.

[8] Bergantiños, G., Moreno-Ternero, J.D., 2021. Compromising to share the revenues from broadcasting sports leagues. Journal of Economic Behavior and Organization 183, 57-74.

[9] Bergantiños, G., Moreno-Ternero, J.D., 2022a. On the axiomatic approach to sharing the revenues from broadcasting sports leagues. Social Choice and Welfare 58, 321-347.

[10] Bergantiños, G., Moreno-Ternero, J.D., 2022b. Monotonicity in sharing the revenues from broadcasting sports leagues. European Journal of Operational Research 297, 338-346.

[11] Bergantiños, G., Moreno-Ternero, J.D., 2022c. Separable rules to share the revenues from broadcasting sports leagues. Economics Letters 211, 110233.

[12] Bergantiños, G., Moreno-Ternero, J.D., 2023a. Decentralized revenue sharing from broadcasting sports. Public Choice 194, 27-44.

[13] Bergantiños, G., Moreno-Ternero, J.D., 2023b. Broadcasting La Liga. Sports Economics Review 1, 100005.

[14] Bergantiños, G., Moreno-Ternero, J.D., 2023c. Anonymity in sharing the revenues from broadcasting sports leagues. Annals of Operations Research. Forthcoming. https://doi.org/10.1007/s10479-023-05345-y

[15] Bergantiños, G., Moreno-Ternero, J.D., 2023d. Broadcasting revenue sharing after cancelling sports competitions. Annals of Operations Research. Forthcoming. https://doi.org/10.1007/s10479-023-05246-0

[16] Birnberg, J.G., Pondy, L.R, Davis, C.L., 1970. Effect of three voting rules on resource allocation decisions. Management Science 16, 356-372.

[17] Brams, S., Fishburn, P., 1978. Approval voting. American Political Science Review 72, 831-847.

[18] van den Brink, R., 2007. Null or nullifying players: The difference between the Shapley value and equal division solutions. Journal of Economic Theory 136, 767-775.

[19] van den Brink, R., Funaki Y., Ju, Y., 2013. Reconciling marginalism with egalitarianism: consistency, monotonicity, and implementation of egalitarian Shapley values. Social Choice and Welfare 40, 693-714.

[20] van den Brink, R., Moreno-Ternero, J.D., 2017. The reverse TAL-family of rules for bankruptcy problems. Annals of Operations Research 254, 449-465.

[21] Casajus, A., Huettner, F., 2013. Null players, solidarity, and the egalitarian Shapley values, Journal of Mathematical Economics 49, 58-61.

[22] Casajus A., Yokote, K., 2019. Weakly differentially monotonic solutions for cooperative games. International Journal of Game Theory 48, 979-997.

[23] Cave, M., Crandall, R., 2001. Sports rights and the broadcast industry. Economic Journal 111, F4-F26.

[24] Dasgupta, P.S., Sen, A.K., Starret, D., 1973. Notes on the Measurement of Inequality. Journal of Economic Theory 6, 180-187.

[25] Falconieri, S., Palomino, F., Sákovics, J., 2004. Collective versus individual sale of television rights in league sports. Journal of the European Economic Association 2, 833-862.

[26] Fort, R., Quirk, J., 1995. Cross subsidization, incentives and outcome in professional team sports leagues. Journal of Economic Literature 33, 1265-1299.

[27] Gans, J. S., Smart, M., 1996. Majority voting with single-crossing preferences. Journal of Public Economics 59, 219-237.

[28] Ginsburgh, V., Moreno-Ternero, J.D., 2023. The Eurovision Song Contest: Voting Rules, Biases and Rationality. Journal of Cultural Economics 47, 247-277.

[29] Ginsburgh, V., Zang, I., 2003. The museum pass game and its value. Games and Economic Behavior 43, 322-325.

[30] Glasser, G., 1959. Game theory and cumulative voting for corporate directors. Management Science 5, 151-156.

[31] Gonçalves-Dosantos J.C., García-Jurado I., Costa J., Alonso-Meijide J.M., 2022. Necessary players and values. Annals of Operations Research 318, 935-961.

[32] Hemming, R., Keen, M.J., 1983. Single-crossing conditions in comparisons of tax progressivity. Journal of Public Economics 20, 373-380.

[33] Hougaard, J.L., Moreno-Ternero, J.D., Østerdal, L.P. 2012. A unifying framework for the problem of adjudicating conflicting claims. Journal of Mathematical Economics 48, 107-114.

[34] Jakobsson, U., 1976. On the measurement of the degree of progression. Journal of Public Economics 5, 161-168.

[35] Marhuenda F., Ortuño-Ortin, I., 1998. Income taxation, uncertainty and stability. Journal of Public Economics 67, 285-300.

[36] Moreno-Ternero J.D., 2011. Voting over piece-wise linear tax methods. Journal of Mathematical Economics 47, 29-36.

[37] Moreno-Ternero, J., Roemer, J., 2006. Impartiality, priority and solidarity in the theory of justice. Econometrica 74, 1419-1427.

[38] Moreno-Ternero, J.D., Vidal-Puga, J. 2021. Aggregator operators for dynamic rationing. European Journal of Operations Research 288, 682-691.

[39] Moreno-Ternero, J.D., Villar, A., 2006. The TAL-family of rules for bankruptcy problems. Social Choice and Welfare 27, 231-249.

[40] van den Nouweland, A., Borm, P., van Golstein Brouwers, W., Groot Bruinderink, R., Tijs, S., 1996. A Game Theoretic Approach to Problems in Telecommunication. Management Science 42, 294-303.

[41] O'Neill, B., 1982. A problem of rights arbitration from the Talmud. Mathematical Social Sciences 2, 345-371.

[42] Peeters, T., 2012. Media revenue sharing as a coordination device in sports leagues. International Journal of Industrial Organization 30, 153-163.

[43] Sawyer, J., MacRae, D., 1962. Game theory and cumulative voting in Illinois: 1902-1954. American Political Science Review 56, 936-946.

[44] Shapley, L., 1953. A value for n-person games, in Contributions to the Theory of Games II (Annals of Mathematics Studies 28), ed. by H.W. Kuhn and A.W. Tucker, Princeton: Princeton University Press, 307-317.

[45] Thomson, W., 2003. Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: a survey, Mathematical Social Sciences 45, 249-297.

[46] Thomson, W. 2011. Consistency and its converse: an introduction. Rev Econ Design 15, 257-291.

[47] Thomson W., 2015. Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: an update. Mathematical Social Sciences 74, 41-59.

[48] Thomson W., 2019. How to divide when there isn't enough: from Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation, Econometric Society Monograph. Cambridge University Press

[49] Thomson, W., Myerson, R. 1980. Monotonicity and independence axioms. International Journal of Game Theory 9, 37-49.

[50] Thomson, W., Yeh, C.H. 2008. Operators for the adjudication of conflicting claims. Journal of Economic Theory 143, 177-198.